Linear Programming Models

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Introduction to Linear Programming

- A Linear Programming model seeks to maximize or minimize a linear function, subject to a set of linear constraints.
- The linear model consists of the following components:
  - A set of decision variables.
  - An objective function.
  - A set of constraints.

Introduction to Linear Programming

- The Importance of Linear Programming
  - Many real world problems can be solved using linear programming modeling.
  - Many real world problems can be approximated by linear models.
  - There are well-known successful applications in:
    - Manufacturing
    - Marketing
    - Finance (investment)
    - Advertising
    - Agriculture

Introduction to Linear Programming

- The Importance of Linear Programming
  - There are efficient solution techniques that solve linear programming models.
  - The output generated from linear programming packages provides useful “what if” analysis.

Introduction to Linear Programming

- Assumptions of the linear programming model
  - Finite Objective Functions. A Linear Programming problem requires a clearly defined, unambiguous objective function which is to be optimized.
  - The parameter values are known with ***certainty.***
  - Linearity Criterion. It is important to ensure that the relationship between decision variables be linear.
  - The Continuity assumption: Variables can take on any value within a given feasible range.
  - Non-Negative Restrictions(decision variables >=0).

Linear programming General Model

Objective function: (Max/Min) $Z = c1\,x1 + \ldots\ldots\ldots\ldots\ldots\ldots + cnxn$

Constraints:

$a11x1 + \ldots\ldots\ldots\ldots\ldots\ldots + a1nxn \leq \geq b1$

$a21x1 + \ldots\ldots\ldots\ldots\ldots\ldots + a2nxn \leq \geq b2$

.

.

.

$am1x1 + \ldots\ldots\ldots\ldots\ldots\ldots + amnxn \leq \geq bm$

$x1, \ldots\ldots\ldots, xn \geq 0$

Where xi are decision variables

Building a linear programming model

The basic steps in formulating a linear programming model are as follows:

Step I. Identification of the decision variables. The decision variables having a bearing on the decision at hand shall first be identified, and then expressed or determined in the form of linear algebraic functions or in equations.

Step II. Identification of the constraints. All the constraints in the given problem which restrict the operation of a firm at a given point of time must be identified in this stage. Further these constraints should be broken down as linear functions in terms of the pre-defined decision variables.

Step III. Identification of the objective. In the last stage, the objective which is required to be optimized (i.e., maximized or minimized) must be clearly identified and expressed in terms of the pre-defined decision variables.

Problem (1)
The Galaxy Industries Production Problem –

- Galaxy manufactures two toy doll models:
  - Space Ray.
  - Zapper.
- Resources are limited to
  - 1000 pounds of special plastic.
  - 40 hours of production time per week.

The Galaxy Industries Production Problem – A Prototype Example

- Marketing requirement
  - Total production cannot exceed 700 dozens.
  - Number of dozens of Space Rays cannot exceed number of dozens of Zappers by more than 350.
- Technological input
  - Space Rays requires 2 pounds of plastic and 3 minutes of labor per dozen.
  - Zappers requires 1 pound of plastic and 4 minutes of labor per dozen.

The Galaxy Industries Production Problem – A Prototype Example

- The current production plan calls for:
  - Producing as much as possible of the more profitable product, Space Ray ($8 profit per dozen).
  - Use resources left over to produce Zappers ($5 profit per dozen), while remaining within the marketing guidelines.

- The current production plan consists of:

Space Rays = 450 dozen

Zapper = 100 dozen

Profit = $4100 per week

8(450) + 5(100)

Management is seeking a production schedule that will increase the company’s profit.

A linear programming model can provide an insight and an intelligent solution to this problem.

The Galaxy Linear Programming Model

- Decisions variables:
  - X_1 = Weekly production level of Space Rays (in dozens)
  - X_2 = Weekly production level of Zappers (in dozens).
- Objective Function:
  - Weekly profit, to be maximized

The Galaxy Linear Programming Model

Max $8X_1 + 5X_2$ (Weekly profit)

subject to

$2X_1 + 1X_2 \leq 1000$ (Plastic)

$3X_1 + 4X_2 \leq 2400$ (Production Time)

$X_1 + X_2 \leq 700$ (Total production)

$X_1 - X_2 \leq 350$ (Mix)

$X_j >= 0, \; j = 1,2$ (Nonnegativity)

2.3 The Graphical Analysis of Linear Programming

The set of all points that satisfy all the constraints of the model is called

a

$\boxed{\text{FEASIBLE REGION}}$

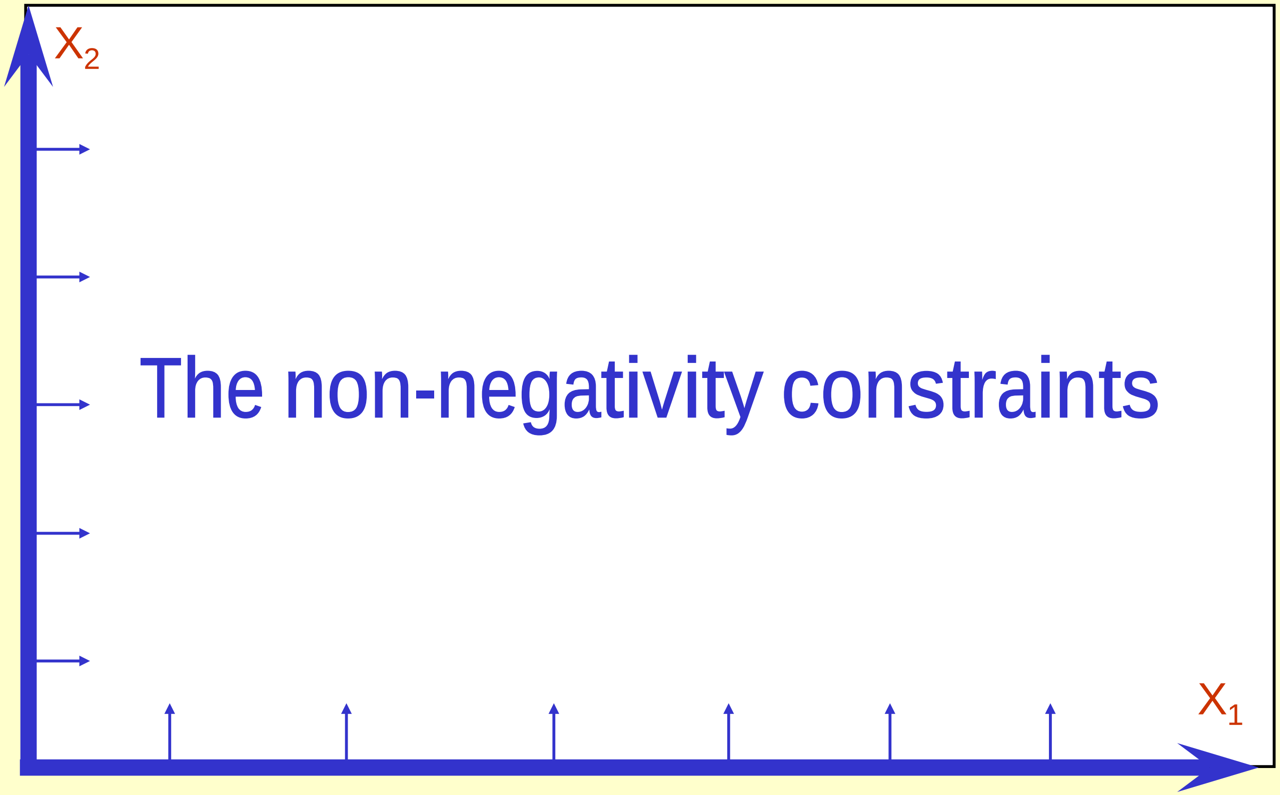
Using a graphical presentation

we can represent all the constraints,

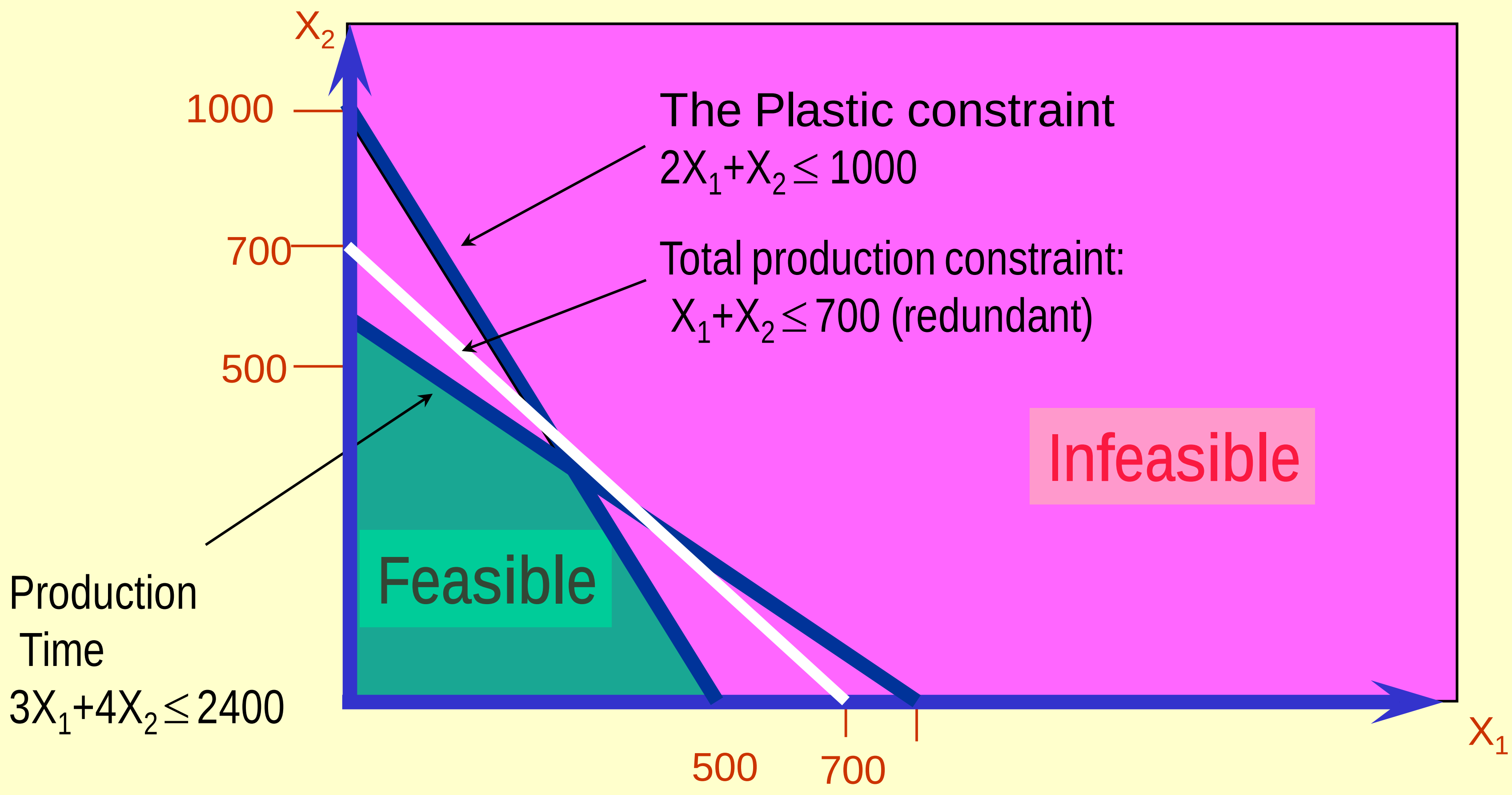
the objective function, and the

types of feasible points.

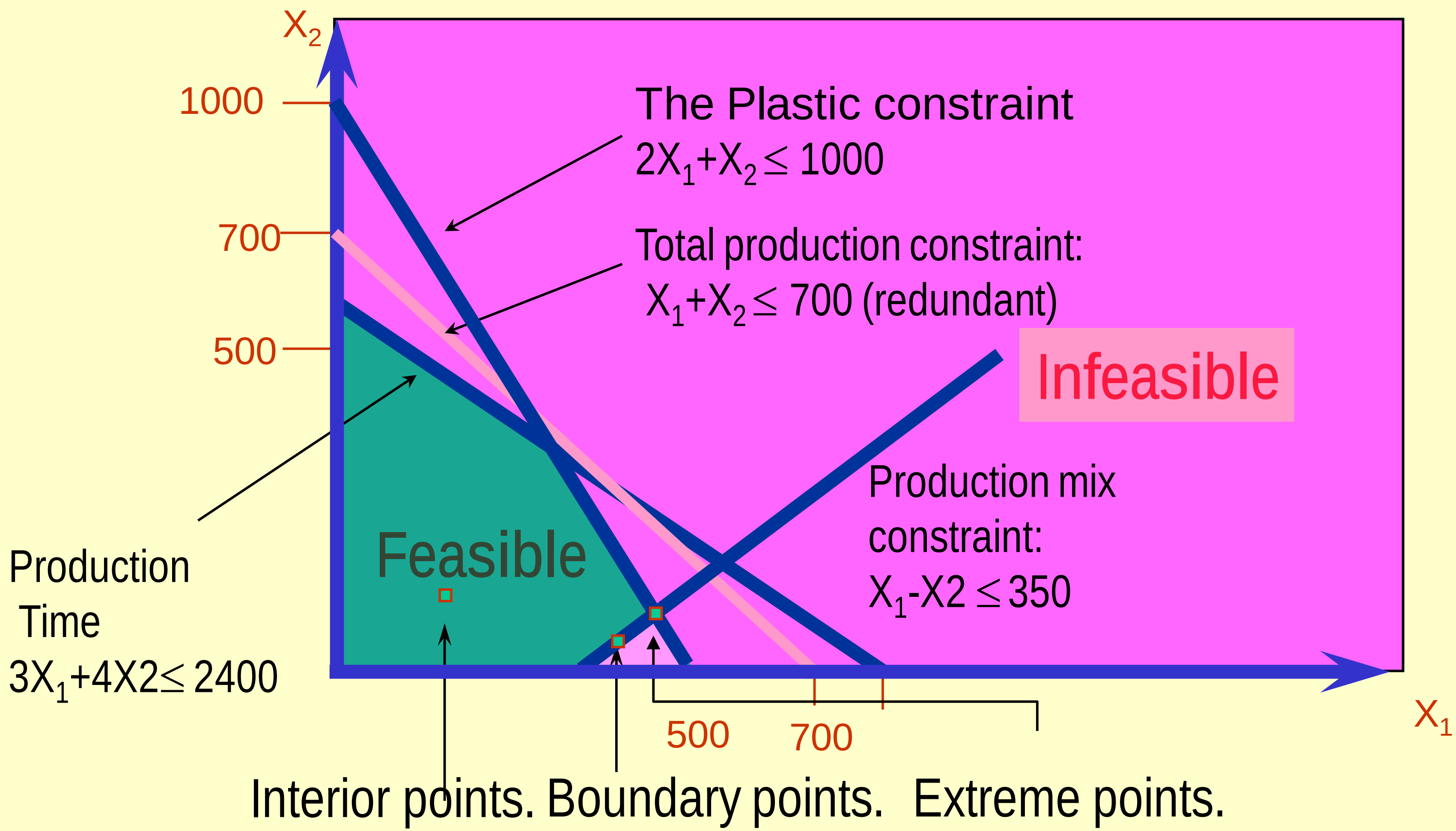
Graphical Analysis – the Feasible Region



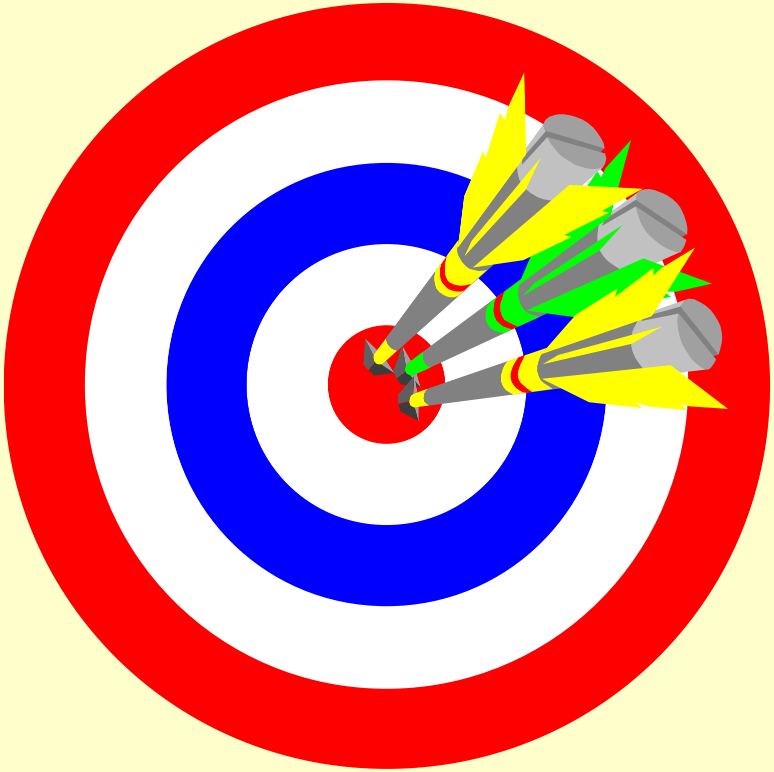
Graphical Analysis – the Feasible Region



Graphical Analysis – the Feasible Region



Solving Graphically for an Optimal Solution

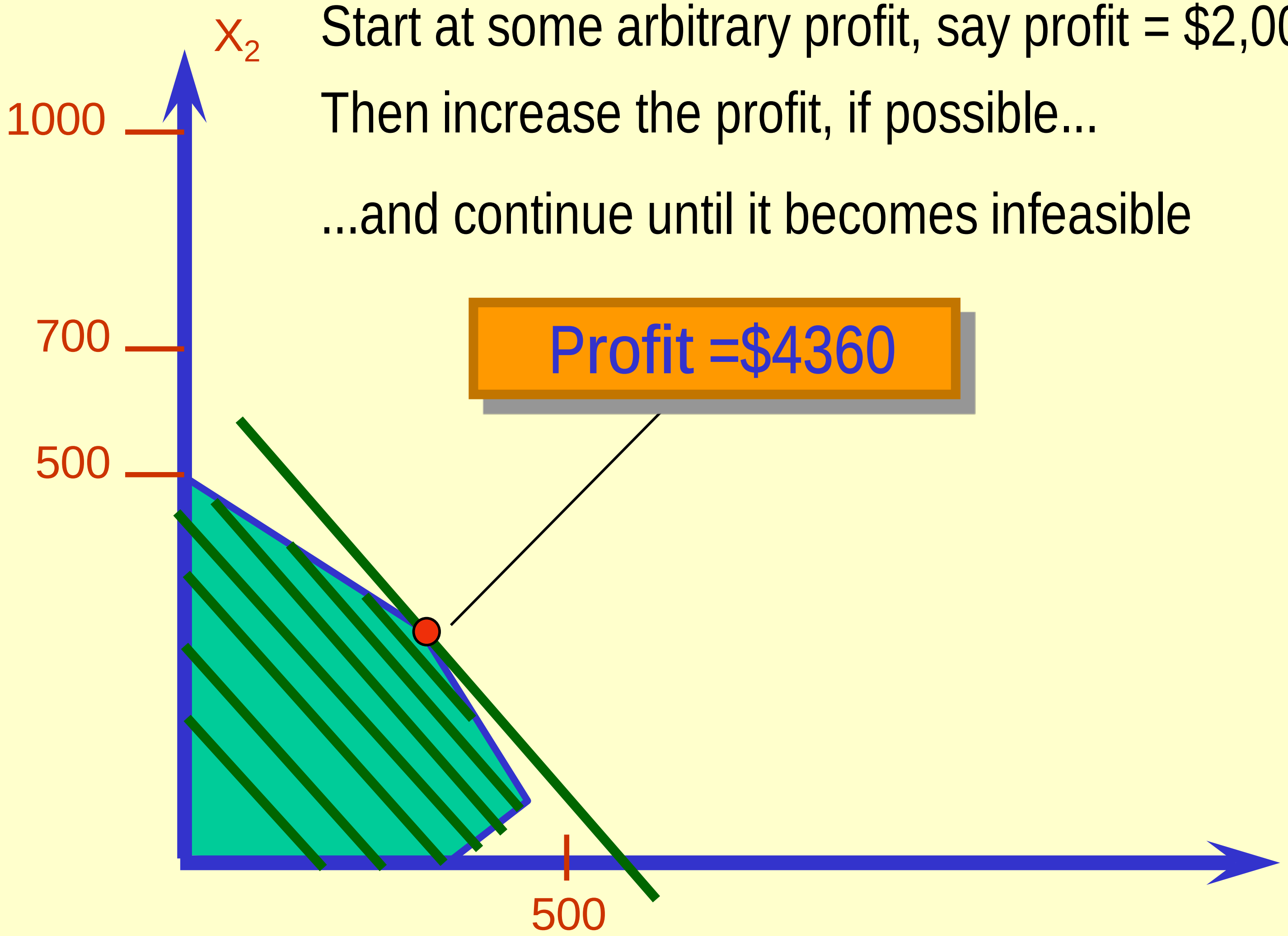


The search for an optimal solution

Start at some arbitrary profit, say profit = $2,000...

Then increase the profit, if possible...

...and continue until it becomes infeasible



Summary of the optimal solution

Space Rays = 320 dozen

Zappers = 360 dozen

Profit = $4360

- This solution utilizes all the plastic and all the production hours.
- Total production is only 680 (not 700).

Other Post - Optimality Changes

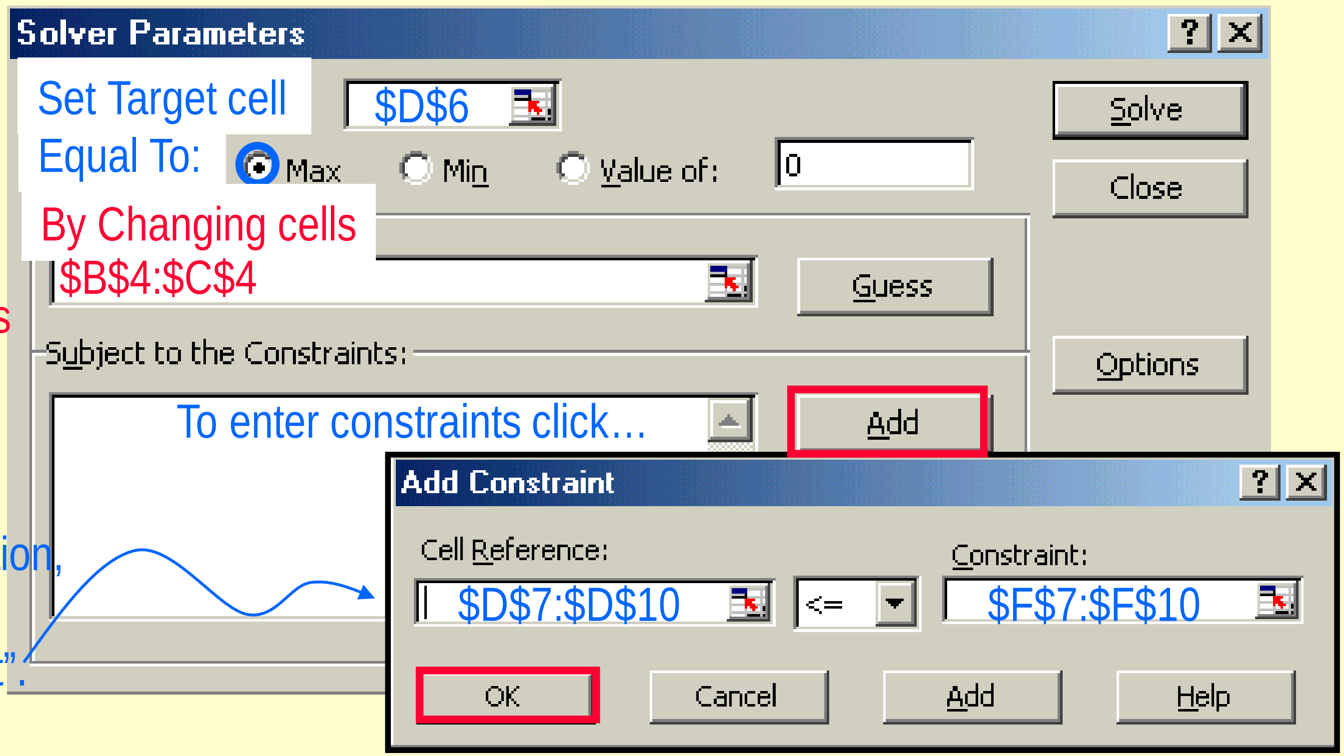
- Addition of a constraint.
- Deletion of a constraint.
- Addition of a variable.
- Deletion of a variable.
- Changes in the left - hand side coefficients.

2.5 Using Excel Solver to Find an Optimal Solution and Analyze Results

- To see the input screen in Excel click Galaxy.xls
- Click Solver to obtain the following dialog box.

This cell contains the value of the objective function

These cells contain the decision variables



All the constraints have the same direction, thus are included in one “Excel constraint”.

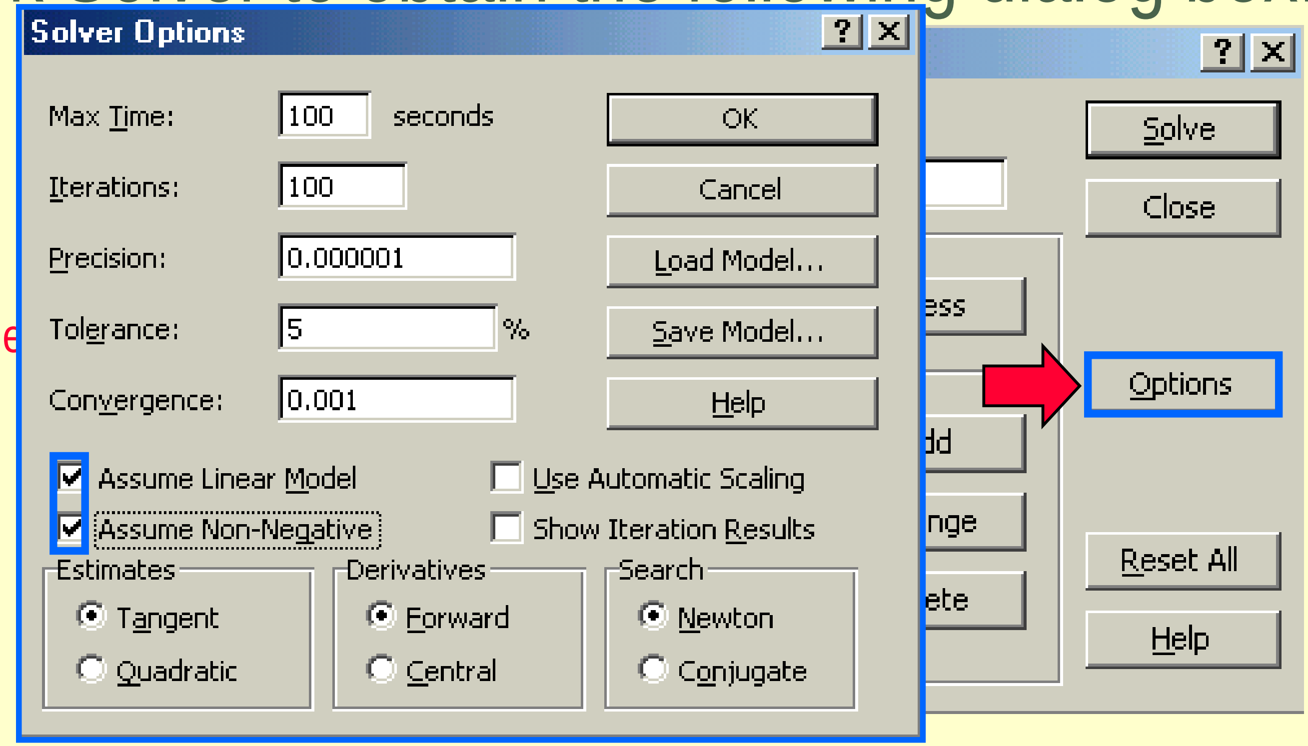
Using Excel Solver

- To see the input screen in Excel click Galaxy.xls
- Click Solver to obtain the following dialog box.

This cell contains the value of the objective function

These cells contain the decision variables

Click on 'Options' and check 'Linear Programming' and 'Non-negative'.



Using Excel Solver

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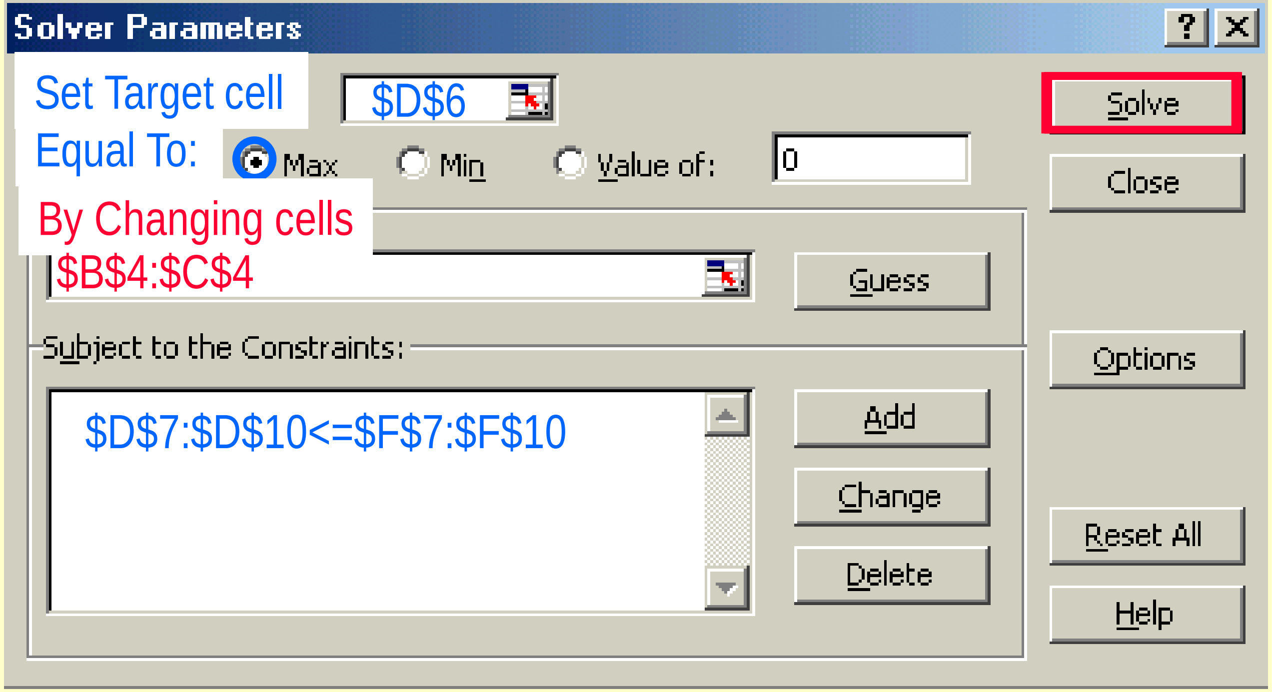
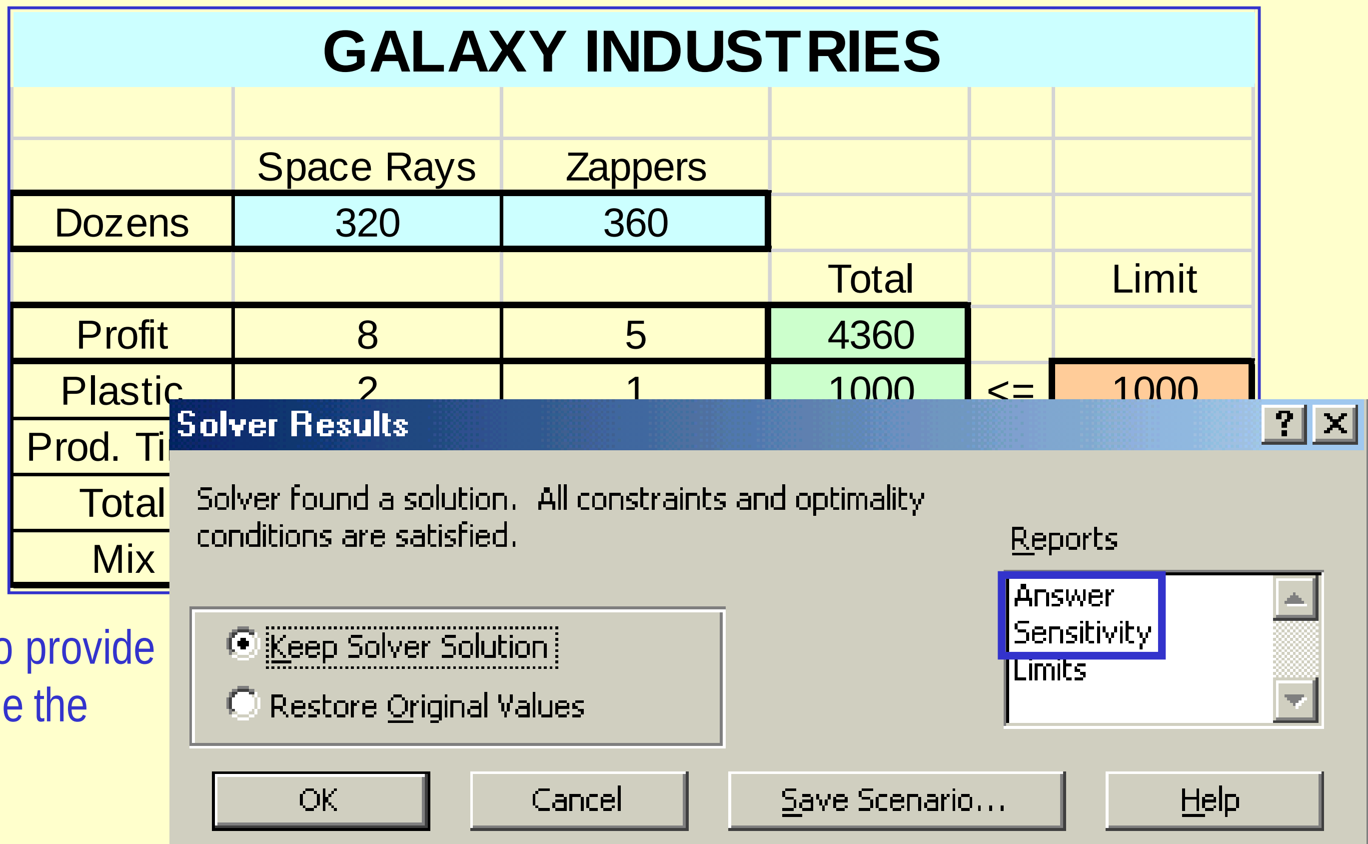


Using Excel Solver – Optimal Solution

GALAXY INDUSTRIES					
	Space Rays	Zappers			
Dozens	320	360			
			Total		Limit
Profit	8	5	4360		
Plastic	2	1	1000	<=	1000
Prod. Time	3	4	2400	<=	2400
Total	1	1	680	<=	700
Mix	1	-1	-40	<=	350

Using Excel Solver – Optimal Solution



Solver is ready to provide reports to analyze the optimal solution.

2.7 Models Without Unique Optimal Solutions

- *Infeasibility*: Occurs when a model has no feasible point.
- *Unboundness:* Occurs when the objective can become infinitely large (max), or infinitely small (min).
- *Alternate solution:* Occurs when more than one point optimizes the objective function

Computer Solution of Linear Programs With Any Number of Decision Variables

- Linear programming software packages solve large linear models.
- Most of the software packages use the algebraic technique called the Simplex algorithm.
- The input to any package includes:
  - The objective function criterion (Max or Min).
  - The type of each constraint: $\leq, =, \geq$.
  - The actual coefficients for the problem.

Problem(2)

SilComputers makes quarterly decisions about their product mix. While their full product line includes hundreds of products, we will consider a simpler problem with just two products: notebook computers and desktop computers. SilComputers would like to know how many of each product to produce in order to maximize profit for the quarter.

There are a number of limits on what SilComputers can produce. The major constraints are as follows:

1. Each computer (either notebook or desktop) requires a Processing Chip. Due to tightness in the market, our supplier has allocated 10,000 such chips to us.

2. Each computer requires memory. Memory comes in 16MB chip sets. A notebook computer has 16MB memory installed (so needs 1 chip set) while a desktop computer has 32MB (so requires 2 chip sets). We received a great deal on chip sets, so have a stock of 15,000 chip sets to use over the next quarter.

Problem(2)

3. Each computer requires assembly time. Due to tight tolerances, a notebook computer takes more time to assemble: 4 minutes versus 3 minutes for a desktop. There are 25,000 minutes of assembly time available in the next quarter.

Given current market conditions, material cost, and our production system, each notebook computer produced generates $750 profit, and each desktop produces $1000 profit.

There are many questions SilComputer might ask. The most obvious are such things as "How many of each type computer should SilComputer produce in the next quarter?" "What is the maximum profit SilComputer can make?" Less obvious, but perhaps of more managerial interest are "How much should SilComputer be willing to pay for an extra memory chip set?" "What is the effect of losing 1,000 minutes of assembly time due to an unexpected machine failure?" "How much profit would we need to make on a 32MB notebook computer to justify its production?"

Linear programming model

$$
\begin{array}{lrcl}
\text{Maximize} & 750x_1 + 1000x_2 & & \\
\text{Subject to} & & & \\
 & x_1 + x_2 & \leq & 10 \\
 & x_1 + 2x_2 & \leq & 15 \\
 & 4x_1 + 3x_2 & \leq & 25 \\
 & x_1 & \geq & 0 \\
 & x_2 & \geq & 0
\end{array}
$$

Optimal Solution

We discussed in the previous section how to solve such equations: the solution here is $x_1 = 1$ and $x_2 = 7$. The optimal decision is to produce 1,000 notebooks and 7,000 desktops, for a profit of \$7,750,000.

End